DETERMINATION OF THE INITIAL TEMPERATURE DISTRIBUTION OF A BODY BY OPTIMAL DYNAMIC FILTERING

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The solution of the converse problem of heat conduction by optimal dynamic filtering using the artificial method of inverse time is considered.

The direct nonstationary problem of heat conduction cannot be solved without fairly complete information on the initial temperature distribution of the body, but in a number of cases this information is not available and the identification of the initial conditions is then of special importance.

The problem of establishing the initial temperature distribution in a body from much later information on its temperature field belongs to the wide class of problems called converse problems. A variety of these problems exist, which, as distinct from inverse and inductive problems, are sometimes called converse problems.

One of the methods which enables converse heat-conduction problems to be solved is the method of the Kalman optimal dynamic filter [1] which has been successfully used to solve heat problems (see, e.g., [2, 3]).

The method of solving the converse problem using an optimal filter consists of three stages.

In the first stage the temperature field is reconstructed from a limited number of measurements. The converse heat-conduction problem is often solved at the same time; i.e., the boundary conditions for heat exchange, if they are unknown, are determined. A finite-difference form of the equations is chosen as the initial condition; these define the thermal state of the object which, in matrix form, characteristic for a discrete filter, has the following form:

$$\mathbf{X}_{k+1} = \Phi_{k+1,k}\mathbf{X}_k + F_{k+1,k}\mathbf{U}_k + G_{k+1,k}\mathbf{W}_k. \tag{1}$$

Here X_{k+1} is the state vector including the temperature-field vector (T_{k+1}) and the vector of the unknown parameters of the boundary conditions (α_{k+1}), $\Phi_{k+1,k}$, $F_{k+1,k}$, $G_{k+1,k}$ are, respectively, the transition matrices of the state, control, and noise vectors.

The algorithm of the discrete Kalman filter used both to reconstruct the temperature field and to solve converse problems of heat conductance, is a recurrent procedure which can be written as follows:

$$\hat{\mathbf{X}}_{k+1|k+1} = \hat{\mathbf{X}}_{k+1|k} + K_{k+1}[\mathbf{Y}_{k+1} - H_{k+1}\hat{\mathbf{X}}_{k+1|k}], \tag{2}$$

$$\hat{\mathbf{X}}_{k+1|k+1} = \{\hat{\mathbf{T}}_{k+1|k+1}, \hat{\boldsymbol{\alpha}}_{k+1|k+1}\},$$

$$\hat{\mathbf{T}}_{k+1|k} = \Phi'_{k+1|k}\hat{\mathbf{T}}_{k|k} + F'_{k+1,k}\mathbf{U}'_k, \tag{3}$$

$$\hat{\boldsymbol{\alpha}}_{k+1|k} = \mathbf{f}(\tau_k), \tag{3'}$$

$$K_{k+1} = P_{k+1|k}H^T_{k+1}[H_{k+1}P_{k+1|k}H^T_{k+1} + R_{k+1}], \tag{4}$$

$$P_{k+1|k} = \Phi_{k+1,k}P_{k|k}\Phi^T_{k+1,k} + G_{k+1,k}Q_kG^T_{k+1,k}, \tag{5}$$

$$P_{k|k} = [I - K_kH_k]\,P_{k|k-1}. \tag{6}$$

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Here $\hat{X}_{k+1/k+1}$ is the optimal estimate of the state vector obtained using the vector of the measurements Y_{k+1}; $\hat{X}_{k+1/k}$ is the prediction of the state vector at the (k + 1)-th instant of time; H_{k+1} is the matrix of measurements, which is related to the measurement and state vectors; $P_{k+1|k}$ and $P_{k|k}$ are covariational matrices of the prediction errors and the errors in estimating the state vector; Q_k and R_{k+1} are the covariational matrices of the random noise at the input of the system (W_k) and the error in the measurements (V_{k+1}) respectively. The transition matrices $\Phi'_{k+1,k}$ and $F'_{k+1,k}$ differ from the corresponding matrices $\Phi_{k+1,k}$ and $F'_{k+1,k}$ in that they contain components of the unknown vector $\boldsymbol{\alpha}$. Note that when solving the direct problem of heat conduction the vector X_k is the vector of the temperature field T_k, and is this case Eq. (3') is eliminated in the algorithm, while the transition matrices $\Phi'_{k+1,k}$ and $F'_{k+1,k}$ must be replaced by the matrices $\Phi_{k+1,k}$ and $F_{k+1,k}$.

To obtain initial estimates of the state vector $\hat{X}_{0/0}$ and the covariational matrix $P_{0|0}$ any a priori information about them can be used.

Since acceptable accuracy (an error of 0.5-0.9%) when solving the direct problem is achieved with the first 5-15 time intervals, the first stage of the solution of the problem can be regarded as completed if the temperature field is obtained in a time interval k > 15.

The second stage is started immediately after the first has ended. In this case the problem is, in effect, turned backwards; i.e., the first time interval of the second stage is the last interval of the first stage, the second interval of the second stage is the penultimate interval of the first etc. We choose as the initial condition in the body the temperature distribution obtained at the last step of the first stage. We take as the reference (measured) values of the temperature those values which are used in the first stage, but the order in which they are treated is, of course, the other way round. The converse problem of heat conduction is solved, in which case the boundary conditions are fictitious since nothing like this occurs in the actual process. For example, the heating process for the converse problem is converted into "cooling," while the cooling process is turned into a "heating" process. As a result, a situation is obtained where the heat-transfer coefficient α becomes negative. Simultaneously with the solution of the converse problem the temperature field is reconstructed in a given time interval which becomes the initial interval for the following step.

The whole system (2)-(6) is used as the algorithm for solving the problem at the second stage. The second stage is completed after the last time-step is reached at which there are reference (measured) values of the temperature. Finally, the last, third, stage is the prediction of the temperature field at the initial instant of time from Eq. (3), taking into account the time interval between the initial instant of time and the instant of time at which the reference values of the temperature occur. The predicted values of the temperature obtained will be an estimate of the initial values of the state vector.

The most accurate initial values of the temperature are obtained at nodes which, when identifying the thermal parameters, are reference nodes (the measurements are taken at these nodes), and also at nodes close to them. At nodes far from the reference nodes the errors increase. Hence, if possible, it is necessary to take as reference nodes as large a number of nodes as possible. This certainly increases the accuracy of the solution of the converse problem.

As an example, we will consider the one-dimensional converse problem of nonstationary heat conduction for an unbounded plate made of a material with the following characteristics: $\lambda = 50-0.03\ T$ (W/m · deg), $a = 1.23 \cdot 10^{-5}-1.05 \cdot 10^{-8}\ T$ (m²/sec), and with the boundary conditions of the third kind

$$\alpha (T - T_c) = -\lambda \frac{\partial T}{\partial n}$$

with a heat-transfer coefficient which varies with time as given by

$$\alpha = 50 + 0.06\tau \ [\mathrm{W/m^2 \cdot deg}]$$

Since the plate is heated symmetrically, its thickness, equal to 0.08 m, is divided in half and we consider a plate of thickness L = 0.04 m with boundary conditions of the third kind on one boundary and zero boundary conditions of the second kind on the second boundary. The temperature of the heating medium T_c = 600°C. The thickness L is divided into five parts with an interval h = 0.01 inside the plate, and h = 0.005 on the boundaries. The time interval $\Delta\tau$ = 30 min.

The estimated initial conditions, obtained by solving the converse problem, are shown in Table 1.

TABLE 1. Estimates of the Initial Temperature Vector for Two Reference Nodes

No. of ref. nodes	No. of nodes					
	1	2	3	4	5	6
2; 4	97,3	99,8	103,7	106,1	107	107
2; 5	94,1	96,9	101,5	104,3	105,6	105,6
3; 4	93,2	94,3	99,1	100,9	102,1	102,1

TABLE 2. Estimates of the Initial Temperature Vector for One Reference Node

No. of ref. nodes	No. of nodes					
	1	2	3	4	5	6
2	96,2	99,8	101,9	105,9	108,9	108,9
3	92,5	95,3	99,9	102,7	104	104
4	90,1	93,4	97,2	100,1	102	102

If we take into account the fact that the standard initial conditions are characterized by a uniform initial temperature distribution $T_1(0) = 100°C$, we see that the solution of the converse problem is fairly accurate (the maximum error does not exceed 7%).

We carried out a wide range of investigations of the practical stability of the solution obtained. In particular, we varied the position and number of the reference points.

Table 1 shows the initial temperature distribution depending on which nodes were taken as the reference nodes (the second and fourth, the second and fifth, the third and fourth). As can be seen, the position of the reference nodes has only a small effect on the estimates obtained. As regards the variation in the number of reference nodes, as might have been expected, the accuracy of the solution of the converse problem decreases as the number of points at which the temperature is measured is reduced (Table 2), but not sufficient to regard these results as incorrect (the error does not exceed 9.9%).

In addition, we investigated the effect of a change in the time interval, which was done both in the first two stages of the solution of the converse problem, and at the prediction stage. In all cases the estimates obtained for the initial temperature distribution (for two-reference points) did not differ from the standard by more than 5-6%.

The correctness of this method in our opinion is a consequence of the particular feature of the algorithm employed, according to which the minimization of the discrepancy function in each time interval assumes minimization of the error between estimates of the initial parameters and their "true" values, which eliminates the possibility of instabilities occurring in the problems considered.

In conclusion, we draw attention to the way in which errors have been taken into account in the initial data (the measurement errors) when solving converse problems. Whereas when solving the inverse problem of heat conduction (identification of the boundary conditions) these errors must be fairly small, since their effect on the solution is quite considerable, in the converse problem the values of the measurement errors play a much lesser role. This is due to the fact that in the converse problem, in the final analysis, the temperature field is determined at the third stage, at which the measurement errors have a much lesser effect, and the "accuracy" of the fictitious boundary conditions obtained is in fact of no interest.

NOTATION

$\hat{X}$, $\hat{T}$, and $\hat{\alpha}$	are the state vectors;
$\hat{W}_k$	is the noise vector;
$\hat{U}_k$	is the control vector;
$\hat{X}_{k+1\|k+1}$, $\hat{T}_{k+1\|k+1}$ and $\hat{\alpha}_{k+1\|k+1}$	are the estimates for the state vector;
$\hat{X}_{k+1\|k}$, $\hat{T}_{k+1\|k}$, $\hat{\alpha}_{k+1\|k}$	are the predictions for the state vectors;

$\Phi_{k+1,k}$, $\Phi'_{k+1,k}$, $F_{k+1,k}$, $F'_{k+1,k}$, $G_{k+1,k}$ are the transition matrices;
$P_{k+1|k+1}$, $P_{k+1|k}$, Q_k, R_{k+1} are the covariant matrices;
K_{k+1} is the weighing matrix;
H_{k+1} is the measurement matrix;
λ is the thermal conductivity;
α is the thermal diffusivity;
I is the unit matrix;
τ is the time;
h is the grid interval.

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OPTIMAL CONTROL OF THE PROCESS OF HEAT TRANSMISSION BETWEEN BODIES IN CONTACT

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Problems of optimal fast-response control of the process of heat transmission between bodies in contact are considered under constraints on the thermoelastic stresses. Analytic expressions are obtained for the control function – the thermal contact resistance.

The process of heat transmission between bodies in contact is characterized by the presence of a thermal resistance in the contact. It is due to the natural roughness of the surfaces in contact and can result in a substantial redistribution of the temperature fields in the materials making contact [1]. The influence of the thermal resistance in the contact on the heat transmission process is twofold: On the one hand, it diminishes the heat flux and therefore results in an increase in the lifetime of the process, and on the other hand, it reduces the temperature drop in the bodies making contact, i.e., results in a diminution in the thermal stress level therein. The dual nature of the influence of the thermal contact resistance on the heat-transmission process permits formulation of an optimal control problem: Find that control (the time dependence of the thermal resistance) which will result in a minimum time in the attainment of the desired result (the target function) and the temperature stresses will hence not exceed a certain quantity governing the strength of the material. The target function can be quite different. For example, the deviation of the mean body temperature from a previously assigned value will not exceed a certain quantity; a definite temperature level will be achieved at a fixed point, etc. Its selection is dictated by specific circumstances. Such problems originate in the design and designation of the exploitational modes of thermal power plants.

A significant number of investigations have been devoted to methods of solving problems on the optimal control of heating solids. The approach developed in [2, 3], whose crux is that compliance with the equality under conditions constraining the thermal stresses is considered equivalent to the condition of realizing an optimal thermal mode, is used below.

1. Let us assume the process of heat transmission between two halfspaces. This problem can be useful if it is necessary to check the process only at times close to the initial time, or when the items making contact are sufficiently massive.

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